Solving equations and systems of equations by graphing

Structure of the lesson plan

Brief summary of the main activities: Students have to collect date about the birth and death rates of their own country by year and they have to illustrate them graphically. They have to analyse the data of the graphs, highlighting the facts that the data show. In order to do this, they have to search particular data on the Internet, they have to be able to illustrate data using graphs, and to be able to interpret them.

Moreover, students have to agree on what kind of authentic problem can be solved by systems of equations. They also have to consider in which subjects they have already applied systems of equations by graphing (e.g. physics – motion exercises; economics – supply and demand, market, budget line, optimal choice).

Main methodologies: The main methodologies used will include collaborative work, frontal lesson and flipped classroom.

Total time: The total time for this learning unit should be 8 hours. However, this may change according to the students' needs.

Competences:

Knowledge:

- Concepts related to functions.
- Graphical illustration of functions, drawing and describing especially linear functions.

- To solve systems of linear equations graphically, we have to draw the graphs for each of the linear equations with the same set of variables. (We have to pay special attention to the fact that the same unknown variable should be considered the independent variable in both of the equations, and the other one should be considered the dependent variable.)
 A system of equations can be solved only if there is a point of intersection between the two graphs. The solution will be the coordinates of the point of intersection, that is, a pair of numbers (x, y). If the graphs intersect in at least two points, there are an infinite number of solutions to the system of equations. If there is no point of intersection, which means that the two graphs are parallel, there is no solution to the system of equations.

- Solving systems of equations by balancing scales.
- The general form of linear systems of equations and solving them by graphs.
- Determining the number of solutions: one pair of numbers (x, y), no solutions; or infinitely many solutions.
- Solving simple problems by graphing.
- Highlighting the importance of checking.

Skills:

- to be able to solve one of the unknown variables
- to be able to illustrate functions graphically, to work precisely
- to be able to check answers algebraically

Soft skills

- Collaborative skills
- Respect towards each other

Learning outcomes:

European Key Competences: that will be developed

1. communication in one's mother tongue;
2. foreign languages;
3. digital skills;
4. basic skills in maths;
5. learning to learn

The product of learning unit:

Prezi Presentation showing the study of solving systems of equations by graphing and the presentation of the study.

Profile of the teacher; terms of reference:

The teachers have to know how to work on *Kahoot*, *Socrative* digital platform, *Prezi* Presentation and how to upload videos.

Assessment:

Prerequisite assessment

- reading graphs and interpreting data they show
- matching functions and assignment rules
- determining the domain and range of functions
- solving simple problems by using systems of equations

How to assess prerequisites – one test of 30 minutes

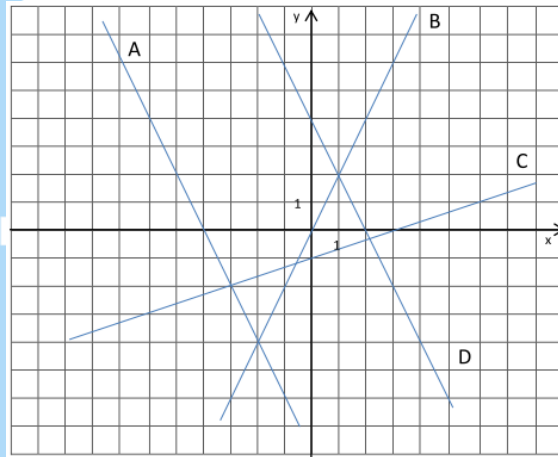10 tasks: you can see the points assigned to tasks in the prerequisite test

Total score: 20 points

Prerequisite test (Total score: 20 points)

1. Choose the correct graph belonging to the function below. (3 points)

$y = 2x$

    A   B   C   D

$y = \dfrac{1}{3}x - 1$

    A   B   C   D

$y = -2x + 4$

    A   B   C   D

2. Decide which of the following points fits the given the line. (1 point)
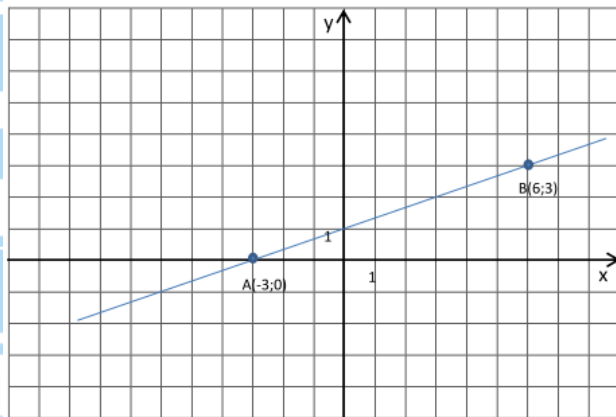
$$y = -\frac{1}{2}x + 3$$

P(10; -1)   Q(6 ; 0)   R(1 ;1)   S(-10 ; 8)

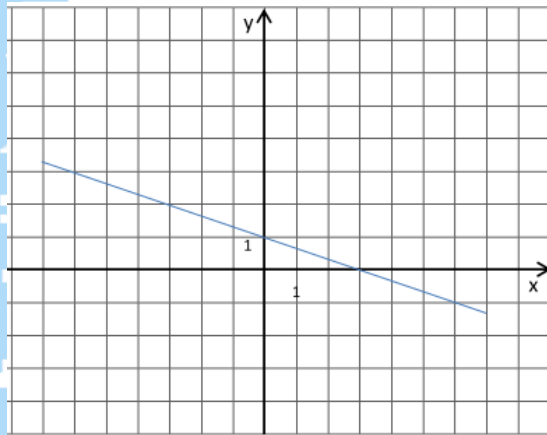3. Describe the equation of the following linear function. (2 points)

4. You can see the graph of a function whose domain is [-4;4]. Find the formula which determines correctly the assignment rule
    for the function below.   (1 point)

A: $y = \dfrac{1}{3}x + 1$

B: $y = -\dfrac{1}{3}x + 1$

C: $y = -3x + 1$

D: $y = -\dfrac{1}{3}x + 3$

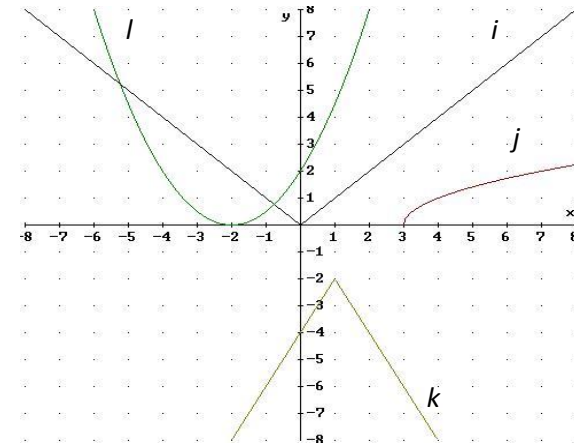5. Match the functions with their assignment rules.   (4 points)

a: $x \mapsto |x|$;

b: $x \mapsto \sqrt{x-3}$;

c: $x \mapsto -2|x-1|-2$

d: $x \mapsto \dfrac{1}{2}(x+2)^2$

a	b	c	d

6. The range of which functions is   ]-∞;-2]?   (1 point)

l)   i)   j)   k)

8

7. Ann and George are travelling to Lake Balaton by car. The graph below shows their covered distance related to time. Having left the town, before driving onto the highway, they stopped at a petrol station in order to refuel their car. Approximately how much time did they spend there? (1 point)
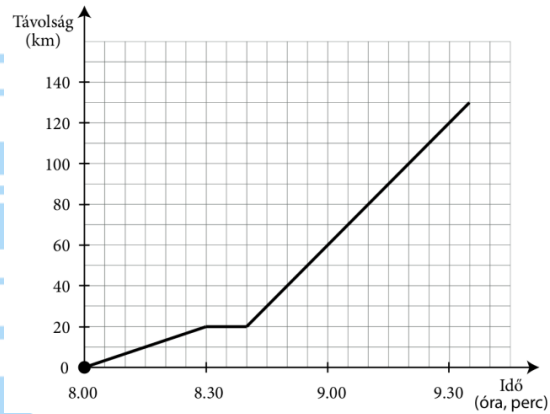
8. Andrew leaves for an observation point at 8 o'clock in the morning. In the first half an hour he walks 5 kilometres, in the second hour he covers only 3 kilometres. Therefore after the second hour he takes a 30-minute break, when he eats his packed lunch from home. After the break he reaches the observation point by 12 o'clock (at noon) maintaining the speed of 4 km/h. (Total: 5 points)

a) How far is Andrew from his village at 9 o'clock? (1 point)

b) How far is the observation point from the village? (2 points)

c) When was Andrew 5 kilometres away from the observation point? (1 point)

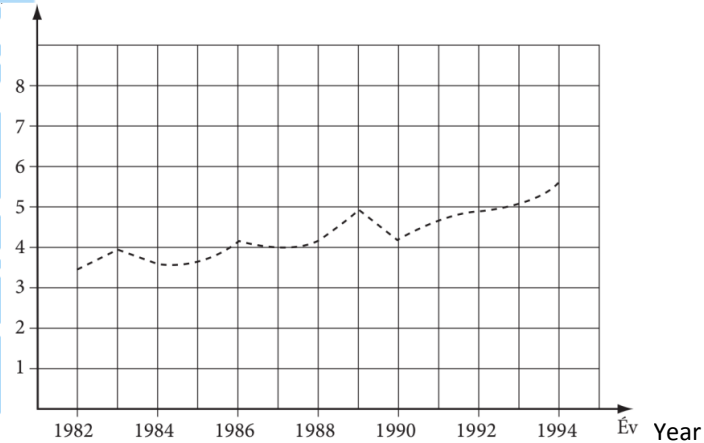d) When was Andrew 10 kilometres away from the observation point? (1 point)

9. The graph below shows the number of people having allergies in Hungary between 1982 and 1994. (1 point)

The number of people having allergies out of 100

Decide which of the following statements is supported by the data of the graph?

A   In 1989, 4% of the people living in Hungary had allergies.
B   In 1990, 4-5% of the people living in Hungary had allergies.
C   Between 1990 and 1991 the number of patients suffering from allergies decreased in Hungary.
D   Between 1989 and 1991 the number of patients suffering from allergies increased in Hungary.

10. The following diagram shows the correlation between the age of a stag (male deer) and the weight of its antlers. Read the diagram and determine how old the stag was when his antlers weighed 5.5 kg.   (1 point)

Final Assessment:

Final test

Time: 60 minutes

❖ 16 tasks

Score for the final test :

You can see the points assigned to each task in the final assessment test.

Total score is **30** points. $x = \frac{score*100}{30}$

| Score	Assessment
$x < 40$	This score range shows little or no skill in responding to the task
$40 \leq x < 60$	This score range demonstrates some developing skill in responding to the task.
$60 \leq x < 70$	This score range demonstrates adequate skill in responding to the task
$70 \leq x < 80$	This score range demonstrates competent skill in responding to the task
$80 \leq x \leq 100$	This score range demonstrates effective skill in responding to the task

Final test (Total: 30 points)

1. The triangle diagram shows that the GDP in Kongo is 44 % of agriculture, 22% of industry and 34% of services. When you draw the diagram, you have to find the point of intersection for the lines starting from the other two axes, moving from the percentage of the scale division lines parallel.
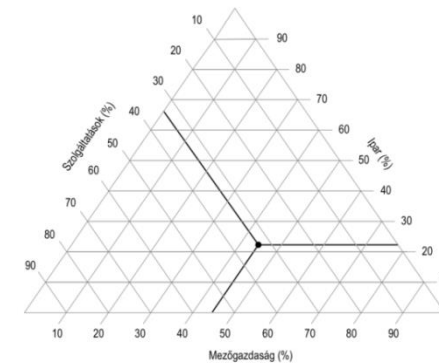
The diagram below shows the economic structure of 4 countries and next to it data of North Korea can be seen.   (1 point)

Észak-Korea	GDP %-a
Mezőgazdaság	23
Ipar	47
Szolgáltatás	30
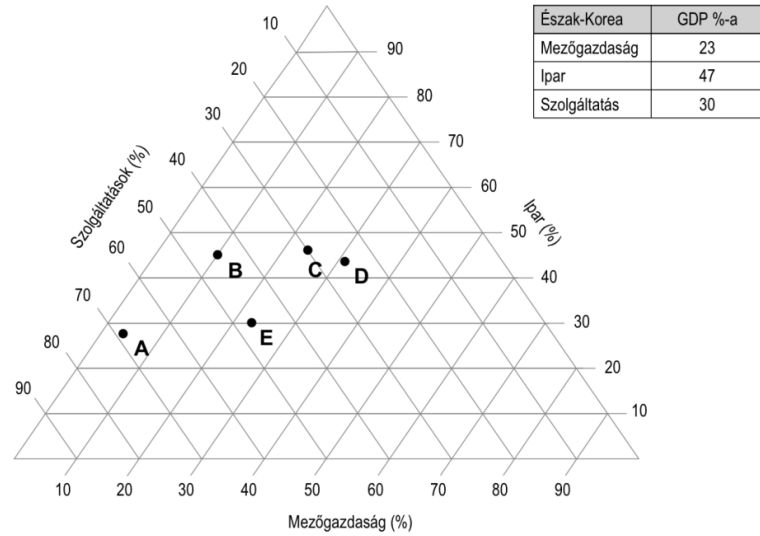
Which point in the diagram illustrates North Korea?
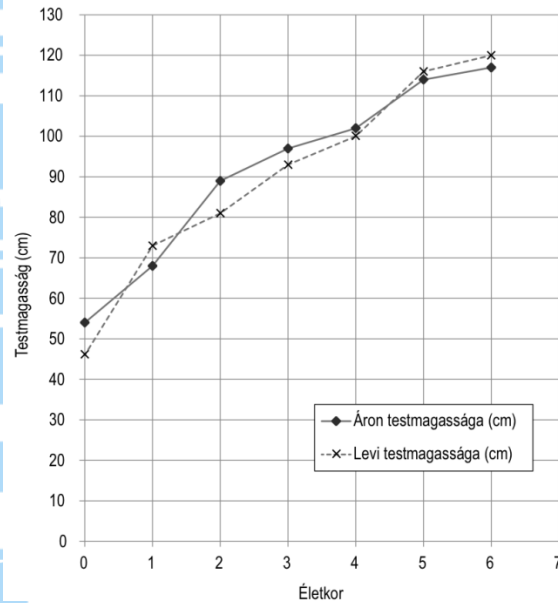
A   B   C   D   E

2. Áron and Levi are twins. Their mother measures their heights on their birthdays every year. The following diagram shows these data.

(1 point)

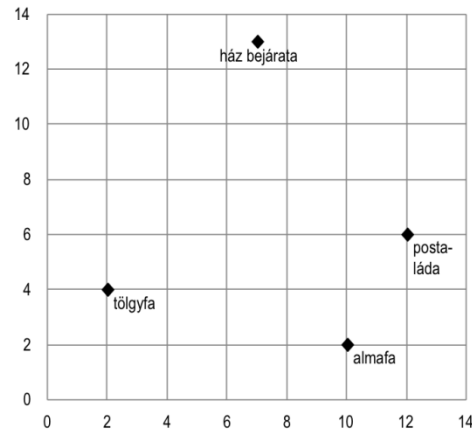Decide which of the following statements is *false*.

A   At the age of 3 Levi was shorter than Áron.

B   By the age of 4 both of them reached the height of 1 m.

C   Áron grew more than Levi until the age of 6.

D   Levi proved to be taller on three occasions of measure than Áron.

3. Sophie dug a treasure box in their garden, drawing a map about the place. (1 point)

ház bejárata: entrance of the house
postaláda: letter box
tölgyfa: oak tree
almafa: apple tree

The treasure box was dug to the same distance from both the oak tree and the apple tree in order to be the same distance from the letter box and the entrance of the house. At which point of coordinate could have she dug the treasure box?

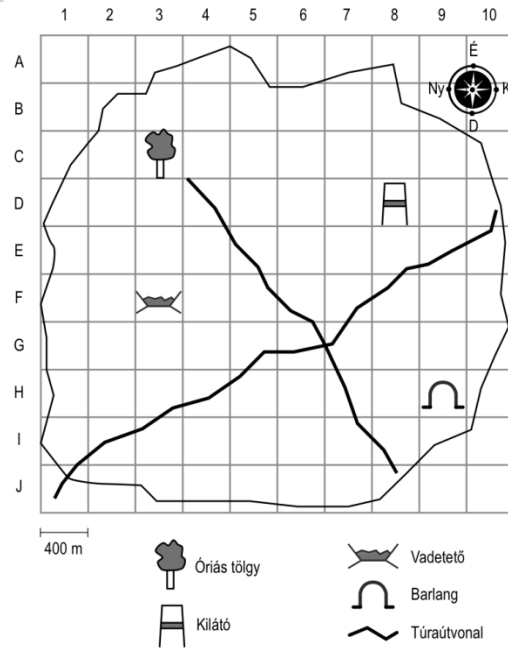A (4;8)   B (7;7)   C (8;8)   D (10;7)

4. You can see the map of a safari park, where a piece of fallen antlers was found. (1 point)

The employees of the safari park found the antlers 600 metres south from the crossing of hiking trails. Determine which field of the map indicates the place of the antlers found.

.... field

5. In the article "Population of Hungary" of a daily newspaper the following figure was published. The graphs show the birth and death rates of the years 1949, 1960, 1970, 1980, 1990, 2001 and 2010.

(1 point)
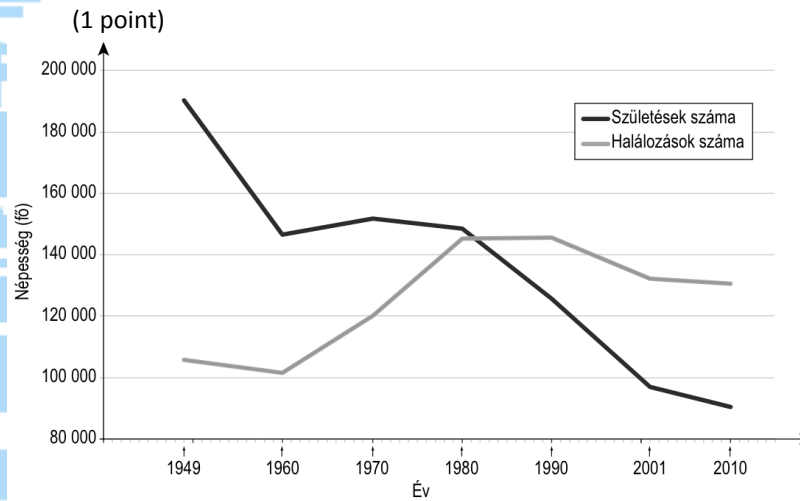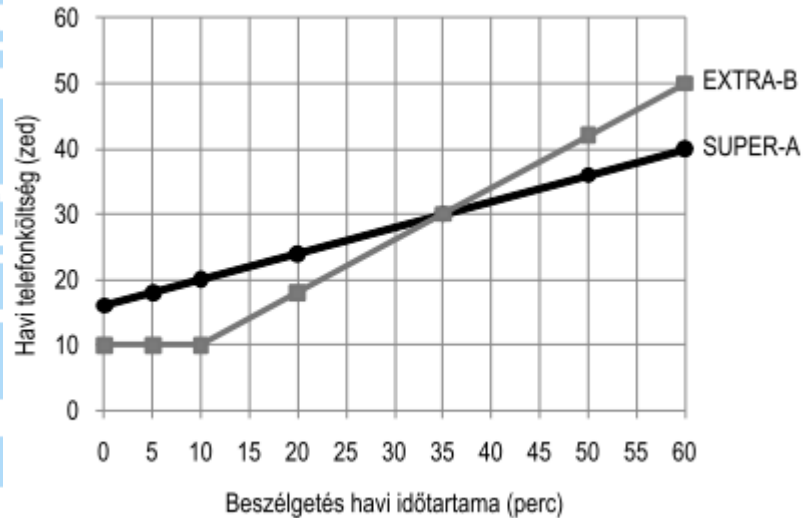

Decide which of the following statements is *true*.

A   After 1983 the death rate was higher than birth rate.

B   In 2010 the death rate was 5 times higher than the birth rate.

C   Between 1949 and 2010 the number of birth was steadily declining.

D   In the years 1970 and 1995 the difference between birth rate and death rate was approximately the same.

6. A company selling mobile phones advertised its special tariff-packages as SUPER-A and EXTRA-B. The cost of the phone bill per month in relation to the amount of time talking per month (in minutes) is shown graphically in the following diagram.
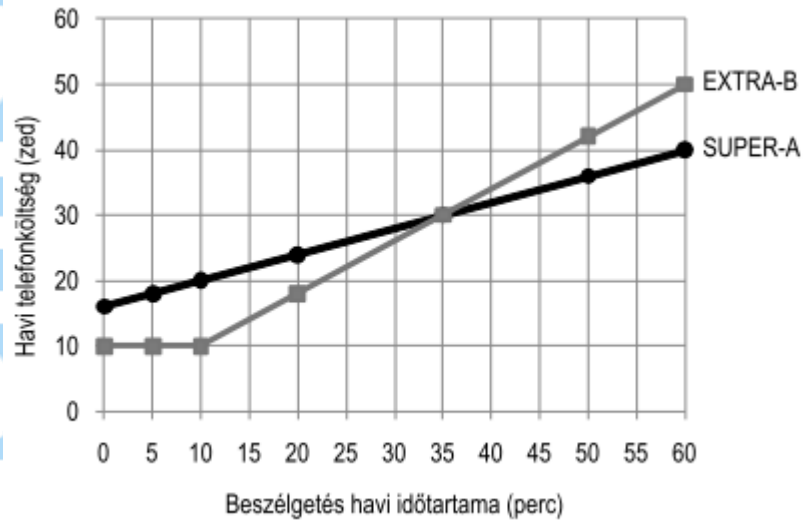
(1 point)

Which of the following statements describes correctly the offer of tariff-package EXTRA-B?

A   The charge per minute of tariff-package for the first 10-minute talk is 10 zed, then exceeding 10 minutes, it increases by 0.4 zed per minute.

B   The charge per minute of tariff-package for the first 10-minute talk is 10 zed, then exceeding 10 minutes, it increases by 0.8 zed per minute.

C   The charge per minute of tariff-package for the first 10-minute talk is 10 zed, then exceeding 10 minutes, it increases by 1.25 zed per minute.

D   The charge per minute of tariff-package for the first 10-minute talk is 10 zed, then exceeding 10 minutes, it increases by 2.5 zed per minute.

7. In October, Barbara talked for 60 minutes with tariff package SUPER-A. However, in the next month, she chose tariff package EXTRA-B instead of EXTRA-A. The cost of her mobile phone bill was 22 zed less than in October.

How many minutes did Barbara talk in November with tariff package EXTRA-B?

(2 points)

8. Swinging ball is a popular game. The aim of the game is to swing the ball hanging from a rope far enough to knock as many bowling pins as possible. Peter was playing with a swinging ball, but as a result of an accidental move, he swung the ball in a way that it hit the pole holding the ball.

(1 point)
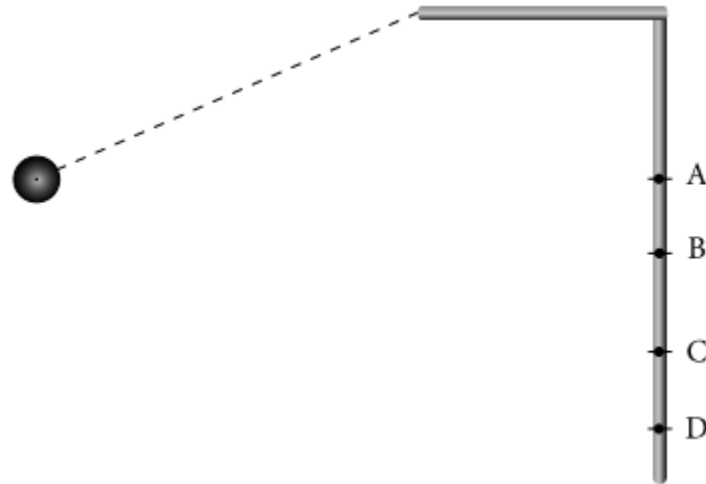
Which point was hit by the ball if the rope was tight all the time?
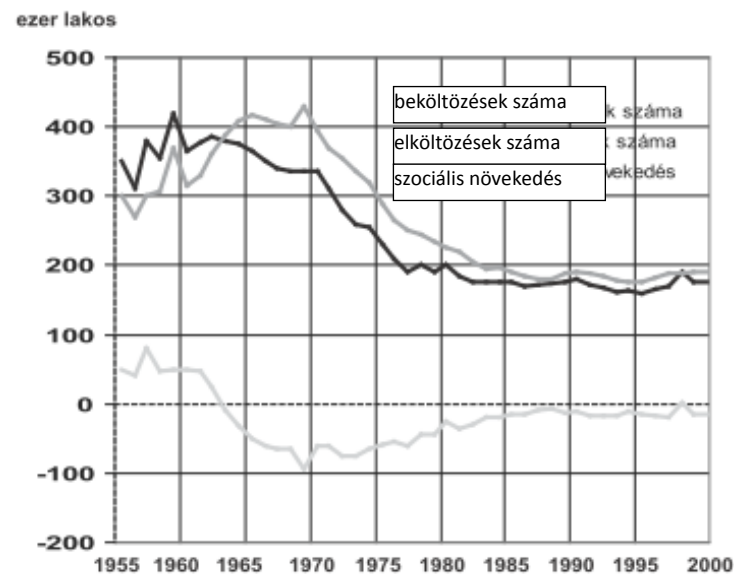
A Point A

B Point B

C Point C

D Point D

9. The graphs below show the number of moving in (black graph) and the number of moving out of a town (grey graph) per year between 1955 and 2000. The third graph illustrates the so-called social growth, that is, the changes resulting from moving in and out of a town.

(1 point)

Is it possible to have a common point for the graph of moving in and that of social growth?

A   Yes, when twice as many people move in as move out.

B   Yes, when nobody moves out.

C   No, because in this case nobody would live in the town.

D   No, because the graph of moving out would fall into the negative domain.

10. In the following graphs you can see the most frequently used routes the so-called air bridges in a coordination system, the centre of which is the airport.
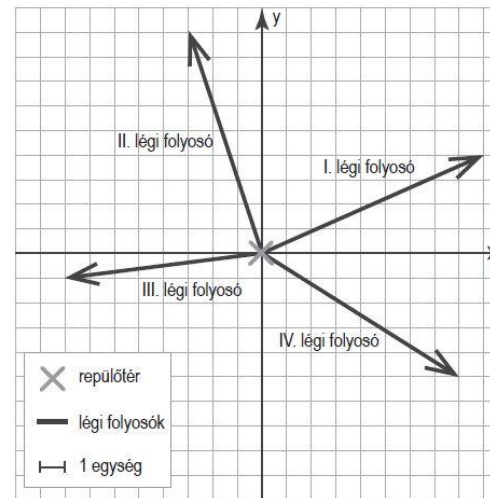
Not far away from the airport a rare bird species nests in the place of (3; 7) coordination. The birds moves to their breeding sites (0; -8) when the cold weather arrives. In which air bridges should the birds be paid more attention to in this season if those fly in straight lines and at the same height as airplanes. Chose the correct answer.
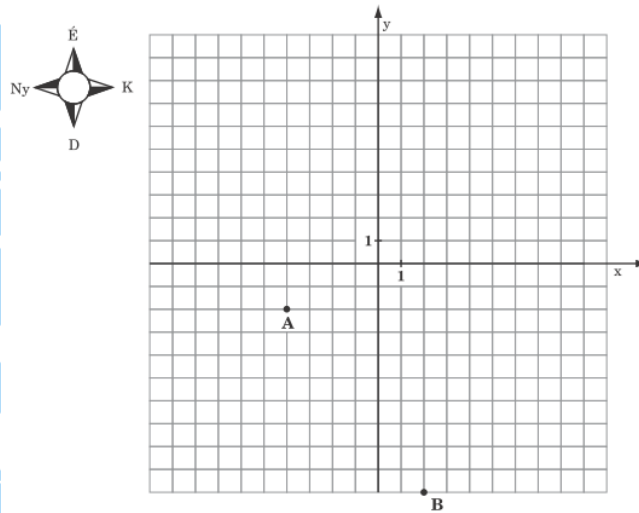
(1 point)

A   In all of the four air bridges.
B   In air bridges I and II.
C   In air bridges II and III.
D   In air bridges I and IV.

11. The plane leaving from town A touching points (4; 6) flies directly to northeast in a straight line. The plane leaving from town B flies to north touching points (2; -1). During their flights there is a point where they usually arrive at about the same time. At around which coordinate point do they have to fly at different heights in order to avoid mid-air collision?
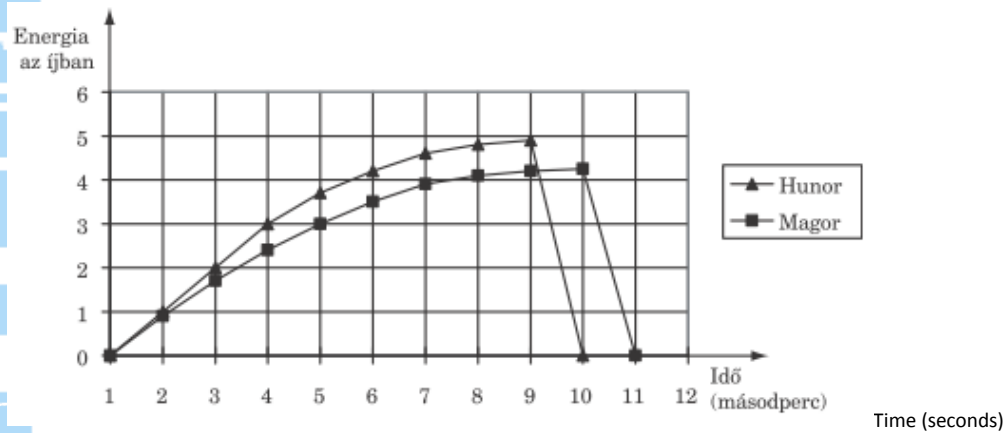
(2 points)

Answer: Around (_____ ; _____) coordinate point.

12. The weapon of land-taking Hungarians was a recurve bow. This kind of bow consists of a flexible wooden frame and a bowstring, the so-called "nerve". When drawing the nerve, energy is stored in the bow. When the nerve is released, this energy is used to do work to push the arrow forward. Hunor and Magor compete against each other in archery. The winner is who shoots farther. Both of them drew their bows with all their strengths, then shot. The graph below shows how much energy was stored in their bows before and after shooting the arrows.
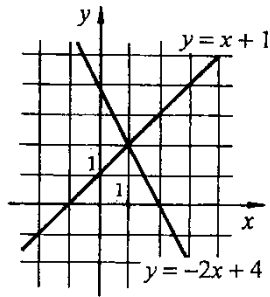
(2 points)

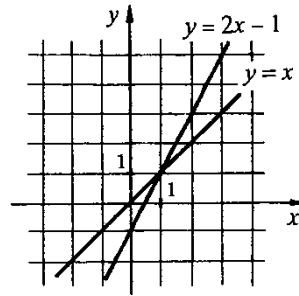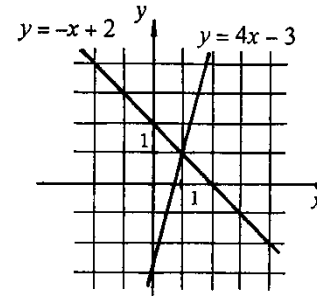Look at the graphs above and decide who shot sooner. Explain why.

13. Look at the graphs and decide which pair of numbers (x; y) solves both of the equations simultaneously.   (4 points)
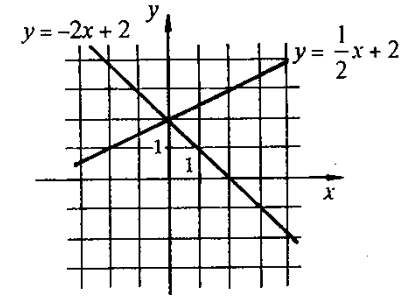

$y = x + 1$

$y = -2x + 4$

$y = 2x - 1$

$y = x$

$y = -x + 2$

$y = 4x - 3$

$y = -2x + 2$

$y = \frac{1}{2}x + 2$

(…..;…..)   (…..;…..)   (…..;…..)   (…..;…..)

14. Solve the following equation systems over the set of real numbers. (3 points)

a)
$$\left.\begin{array}{l} 2x - y = 5 \\ x + y = 4 \end{array}\right\}$$

b)
$$\left.\begin{array}{l} 4x - 3y = -2 \\ -8x + 6y = 4 \end{array}\right\}$$

c)
$$\left.\begin{array}{l} 3x - y = 1 \\ -3x + y = 3 \end{array}\right\}$$

Determine the number of solutions.
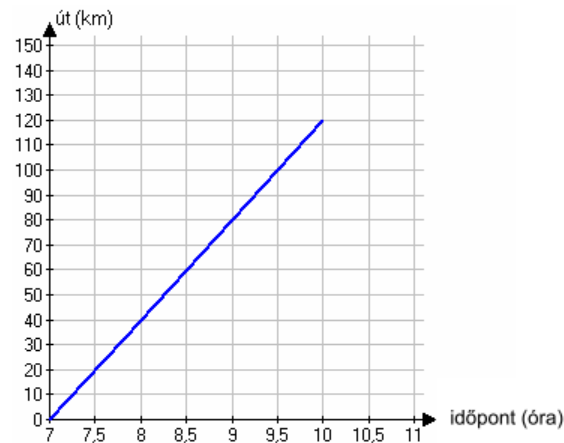
1 solution	no solution	infinitely many

Erasmus+

15. At 7 o'clock a.m. a cargo train leaves from Budapest to Debrecen, which goes without stopping at a uniform speed. In the coordinate system you can see the covered distance by the cargo train in relation to time.

(Total:6 points)

a) How long distance did the cargo train cover in the first hour?   (1 point)
b) Calculate how many hours it takes for the cargo train to cover 108 kilometres.   (2 points)

At 7.30 a.m. an express train leaves from Budapest to Debrecen on the same route. The train moves at a uniform speed of 70 km/h without stopping.

c) Draw the graph of distance-time for the express train in the coordination system above between the time interval of 7.30 and 9.30.   (1 point)
d, Calculate when and after how long distance the express train catches the cargo train.

(2 points)

16. We light two candles of the same height simultaneously. The first candle burns down in 4 hours, the second in 3 hours. (as can be seen on graph I and graph II) Graph III shows the burning down of a candle which is twice as high as the second one.

(2 points)
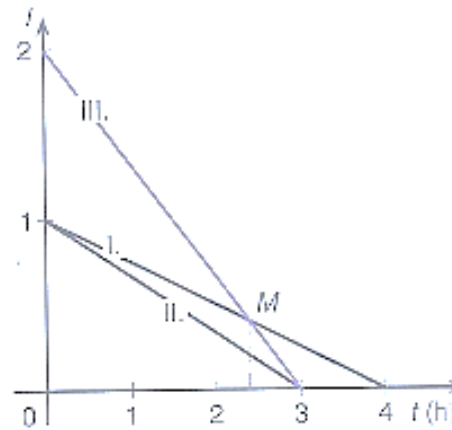
In how many hours will be the first candle-end twice as high as the second one if the height of the candles is decreasing evenly.

A   In 1 hour

B   In 2 hours

C   In 2.4 hours

D   In 3 hours

POCKET Tools
Math – Unit 3

MODULE ARTICULATION

Time	Goal(s)	Learning Activities	Teacher' & Students' Roles	1) Learning Environment 2) Digital Technologies 3) Collaboration/Individual work 4) Reflection /Assessment
1h	To assess prerequisites			
1h	• To propose an authentic problem • To motivate students to use mathematics as a means to make decisions or to solve problems • To activate previous knowledge. • To share ideas and to listen to others	• To propose an authentic problem: **You have to collect data about the birth and death rates of your own country by year and illustrate them graphically. Then you have to analyse the data of the graphs, highlighting the facts that the graphs show.** • The teacher asks students about the criteria to be followed in order to make a decision: This step creates expectation.	**The teacher**: facilitator; controls the time; ensures that all students take part in the activities; asks some questions; activates students' curiosity. **The students:** suggest ideas and strategies in the classroom and they detect data by using various websites.	1. 10 min brainstorming and 15 min for searching data on the Internet, and completing the table below (35 minutes). | Years	1940	…	2016
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Death rate			
Birth rate				 2. Computer or smartphone 3. Collaborative and individual work 4. Informal observation of the group work of students
	HOMEWORK: to search YouTube tutorial videos for solving equation systems by graphing, and for their application in physics and economics			

2h	• To present the steps for the graphical solution of a linear system of equations in which there are two unknowns. • Number of solutions: - One solutions - No solution - Infinite number of solutions To answer the following questions: - What does it mean to solve equation systems by graphing? - How precise/reliable is it? - In which equation systems can it be applied? - Which steps are needed to solve equation systems by graphs? - What determines the number of solutions - Why is checking of great importance? • To use them in simple exercises	• To see videos • To summarise and write the possible solutions on the blackboard and in students' exercise books	**The students** show the videos and explain their doubts; they also develop exercises in their exercise books in pairs or write their solutions on the blackboard. **The teacher** writes the exercises suggested and if necessary the solutions as well on the blackboard. **The teacher** is a guide, advisor, stimulator and observer.	1) 20 min video, 100 min exercises Computer, projector, textbook and exercise book 2) Flipped classroom and pairwork 3) Informal observation of group work. It is also possible to create a rubric of soft skills which can provide a basis for evaluation.

➕ **HOMEWORK:** simple but graduated exercises from the textbook				
1h	• To consolidate knowledge through application • To make learning enjoyable for students	▪ Correction of exercises ▪ Competition with *Kahoot*	**The teacher** designs a test with *Kahoot* and then uses it; helps the students and explans the solutions if necessary. **The students** do exercises on the blackboard and use *Kahoot*.	1) 30 min correction and 30 min competition 2) Textbook, computer, mobile phone 3) Individual work, and team work 4) Informal assessment: tests with *Kahoot*
➕ **HOMEWORK:** graduated exercises from the textbook				
1h	• Retrieval of knowledge	▪ Exercises solved in teams	**The teacher** divides the students into teams of four members of different levels; the students work together. The teacher goes around in order to answer any possible questions. If the same problems come up, the teacher solves them on the blackboard.	1) The whole time 2) Textbook, exercise book 3) Cooperative learning 4) Informal observation of group work

			The students help each other to solve the exercises.	
1h	• To assess knowledge and abilities	▪ Final test	**The students** take the final test created with *Socrative.*	1) 60 min for the test 2) Computers 3) Individual work 4) Final assessment

➕ **HOMEWORK:** to analyse an authentic problem

1h	• To teach students how to discuss their own mistakes • To find out the solution • To develop self-assessment • To share ideas, to listen and to negotiate.	▪ To comment on the test results ▪ To discuss together the solution of the authentic problem	**The teacher** analyses the final test answers with each student. **The students** discuss the solution of the authentic problem in teams. Then one of them writes the solution on the blackboard helped by all the students.	1) 30 min for the analysis of final test 2) 30 min whole-class discussion of the solution of the authentic problem 3) Blackboard, coloured chalks 4) Collaborative and individual work

HOMEWORK: To create a presentation on Learning Unit 3 with *Prezi* and to introduce it in teams. Each team is responsible for the introduction of a certain part of the presentation.